Series EF1GH

SET~4

प्रश्न–पत्र कोड
Q.P. Code **465**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

व्यावहारिक गणित
APPLIED MATHEMATICS

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निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*
Time allowed : 3 hours *Maximum Marks : 80*

नोट / NOTE :

(i) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।*
*Please check that this question paper contains **23** printed pages.*

(ii) *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) *कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।*
*Please check that this question paper contains **38** questions.*

(iv) ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
Please write down the serial number of the question in the answer-book before attempting it.

(v) *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र में* ***38*** *प्रश्न हैं । **सभी** प्रश्न **अनिवार्य** हैं ।*

(ii) *यह प्रश्न-पत्र* ***पाँच*** *खण्डों में विभाजित है – **क, ख, ग, घ** एवं **ङ** ।*

(iii) ***खण्ड क*** *में प्रश्न संख्या* ***1*** *से* ***18*** *तक बहुविकल्पीय तथा प्रश्न संख्या* ***19*** *एवं* ***20*** *अभिकथन एवं तर्क आधारित* ***एक-एक*** *अंक के प्रश्न हैं ।*

(iv) ***खण्ड ख*** *में प्रश्न संख्या* ***21*** *से* ***25*** *तक अति लघु-उत्तरीय (VSA) प्रकार के* ***दो-दो*** *अंकों के प्रश्न हैं ।*

(v) ***खण्ड ग*** *में प्रश्न संख्या* ***26*** *से* ***31*** *तक लघु-उत्तरीय (SA) प्रकार के* ***तीन-तीन*** *अंकों के प्रश्न हैं ।*

(vi) ***खण्ड घ*** *में प्रश्न संख्या* ***32*** *से* ***35*** *तक दीर्घ-उत्तरीय (LA) प्रकार के* ***पाँच-पाँच*** *अंकों के प्रश्न हैं ।*

(vii) ***खण्ड ङ*** *में प्रश्न संख्या* ***36*** *से* ***38*** *प्रकरण अध्ययन आधारित* ***चार-चार*** *अंकों के प्रश्न हैं ।*

(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है । यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 2 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 3 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है ।*

(ix) *कैल्कुलेटर का उपयोग **वर्जित** है ।*

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है ।

1. $(22)^{12}$ का आखिरी (इकाई का) अंक है :

(a) 2 (b) 4

(c) 6 (d) 8

2. 3^{15} को 7 से भाग करने पर न्यूनतम ऋणेत्तर शेषफल है :

(a) 1 (b) 5

(c) 6 (d) 7

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The last (unit) digit of $(22)^{12}$ is :

(a) 2 (b) 4

(c) 6 (d) 8

2. The least non-negative remainder, when 3^{15} is divided by 7 is :

(a) 1 (b) 5

(c) 6 (d) 7

3. यदि $A=\begin{bmatrix}1 & 0\\ -2 & 1\end{bmatrix}$ तथा $B=\begin{bmatrix}-5 & 10\\ -10 & -5\end{bmatrix}$ है, तो AB है :

(a) $\begin{bmatrix}-5 & 10\\ 0 & -5\end{bmatrix}$ (b) $\begin{bmatrix}0 & -5\\ 25 & 10\end{bmatrix}$

(c) $\begin{bmatrix}10 & -25\\ -5 & 0\end{bmatrix}$ (d) $\begin{bmatrix}-5 & 10\\ 0 & -25\end{bmatrix}$

4. यदि $\begin{bmatrix}x+y & x+2\\ 2x-y & 16\end{bmatrix}=\begin{bmatrix}8 & 5\\ 1 & 3y+1\end{bmatrix}$ है, तो x तथा y के मान हैं :

(a) $x=3, y=5$ (b) $x=5, y=3$

(c) $x=2, y=7$ (d) $x=7, y=2$

5. वह अनुपात जिसमें एक दुकानदार दो प्रकार की दालों, जिनके मूल्य क्रमशः ₹ 85 प्रति किग्रा तथा ₹ 100 प्रति किग्रा हैं, को मिलाकर ₹ 92 प्रति किग्रा का मिश्रण प्राप्त करता है, है :

(a) 7 : 8 (b) 8 : 7

(c) 5 : 7 (d) 7 : 5

6. यदि सभी $x \in \mathbb{R}$ के लिए, $\frac{|x+1|}{x+1} > 0$ है, तो :

(a) $x \in [-1, \infty)$ (b) $x \in (-1, \infty)$

(c) $x \in (-\infty, -1)$ (d) $x \in (-\infty, -1]$

3. If $A=\begin{bmatrix}1 & 0\\ -2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-5 & 10\\ -10 & -5\end{bmatrix}$, then AB is :

(a) $\begin{bmatrix}-5 & 10\\ 0 & -5\end{bmatrix}$ (b) $\begin{bmatrix}0 & -5\\ 25 & 10\end{bmatrix}$

(c) $\begin{bmatrix}10 & -25\\ -5 & 0\end{bmatrix}$ (d) $\begin{bmatrix}-5 & 10\\ 0 & -25\end{bmatrix}$

4. If $\begin{bmatrix}x+y & x+2\\ 2x-y & 16\end{bmatrix}=\begin{bmatrix}8 & 5\\ 1 & 3y+1\end{bmatrix}$, then the values of x and y are :

(a) $x = 3, y = 5$ (b) $x = 5, y = 3$

(c) $x = 2, y = 7$ (d) $x = 7, y = 2$

5. The ratio in which a grocer mixes two varieties of pulses costing ₹ 85 per kg and ₹ 100 per kg respectively so as to get a mixture worth ₹ 92 per kg, is :

(a) 7 : 8 (b) 8 : 7

(c) 5 : 7 (d) 7 : 5

6. If $\frac{|x+1|}{x+1} > 0, x \in \mathbb{R}$, then :

(a) $x \in [-1, \infty)$ (b) $x \in (-1, \infty)$

(c) $x \in (-\infty, -1)$ (d) $x \in (-\infty, -1]$

7. A तथा B दोनों कोटि 3 के ऐसे वर्ग आव्यूह हैं जिनके लिए $|A| = -1$ तथा $|B| = 3$ है । $|3AB|$ का मान क्या है ?

(a) -9 (b) -18

(c) -27 (d) -81

8. यदि $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$ है, तो x का मान है :

(a) -1 (b) 0

(c) 1 (d) 3

9. किसी उत्पाद की 'x' इकाइयों के उत्पादन के लिए 'सीमांत लागत' (MC) तथा 'माध्य लागत' (AC) में संबंध है :

(a) $\frac{d(AC)}{dx} = x(MC - AC)$ (b) $\frac{d(AC)}{dx} = x(AC - MC)$

(c) $\frac{d(AC)}{dx} = \frac{1}{x}(AC - MC)$ (d) $\frac{d(AC)}{dx} = \frac{1}{x}(MC - AC)$

10. $\int (x-1)e^{-x}\,dx$ बराबर है :

(a) $(x-2)\,e^{-x} + C$ (b) $xe^{-x} + C$

(c) $-xe^{-x} + C$ (d) $(x+1)\,e^{-x} + C$

11. अवकल समीकरण $\frac{dx}{x} + \frac{dy}{y} = 0$ का हल है :

(a) $\frac{1}{x} + \frac{1}{y} = C$ (b) $xy = C$

(c) $\log x \log y = C$ (d) $x + y = C$

7. A and B are square matrices each of order 3 such that $|A| = -1$ and $|B| = 3$. What is the value of $|3AB|$?

(a) -9 (b) -18

(c) -27 (d) -81

8. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is :

(a) -1 (b) 0

(c) 1 (d) 3

9. The relation between 'Marginal cost' and 'Average cost' of producing 'x' units of a product is :

(a) $\frac{d(AC)}{dx} = x(MC - AC)$ (b) $\frac{d(AC)}{dx} = x(AC - MC)$

(c) $\frac{d(AC)}{dx} = \frac{1}{x}(AC - MC)$ (d) $\frac{d(AC)}{dx} = \frac{1}{x}(MC - AC)$

10. $\int (x-1)e^{-x}\,dx$ is equal to :

(a) $(x-2)\,e^{-x} + C$ (b) $xe^{-x} + C$

(c) $-xe^{-x} + C$ (d) $(x+1)\,e^{-x} + C$

11. The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is :

(a) $\frac{1}{x} + \frac{1}{y} = C$ (b) $xy = C$

(c) $\log x \log y = C$ (d) $x + y = C$

12. यदि X एक ऐसा प्वासों चर है कि $P(X = 1) = 2P(X = 2)$, तो $P(X = 0)$ है :

(a) e (b) $\frac{1}{e}$

(c) 1 (d) e^2

13. यदि $|t|$ का परिकलित मान $< t_v(\alpha)$ है, तब निराकरणीय परिकल्पना :

(a) अस्वीकार की जाती है

(b) स्वीकार की जाती है

(c) निर्धारित नहीं की जा सकती

(d) न स्वीकार की जाती है और न ही अस्वीकार

14. दो स्वतंत्र नमूनों के माध्यों के बीच के अंतर की सार्थकता की जाँच करने के लिए स्वातंत्र्य कोटि (v) ली जाती है :

(a) $n_1 - n_2 + 2$ (b) $n_1 - n_2 - 2$

(c) $n_1 + n_2 - 2$ (d) $n_1 + n_2 - 1$

15. रैखिक प्रवृत्ति जिस समीकरण से निरूपित की जाती है, वह है :

(a) $y_c = a + bx$ (b) $y_c = a - bx$

(c) $y_c = na + b\Sigma x$ (d) $y_c = na - b\Sigma x$

12. If X is a Poisson variable such that $P(X = 1) = 2P(X = 2)$, then $P(X = 0)$ is :

(a) e (b) $\frac{1}{e}$

(c) 1 (d) e^2

13. If the calculated value of $|t| < t_v(\alpha)$, then the null hypothesis is :

(a) rejected

(b) accepted

(c) cannot be determined

(d) neither accepted nor rejected

14. For testing the significance of difference between the means of two independent samples, the degree of freedom (v) is taken as :

(a) $n_1 - n_2 + 2$ (b) $n_1 - n_2 - 2$

(c) $n_1 + n_2 - 2$ (d) $n_1 + n_2 - 1$

15. The straight line trend is represented by the equation :

(a) $y_c = a + bx$ (b) $y_c = a - bx$

(c) $y_c = na + b\Sigma x$ (d) $y_c = na - b\Sigma x$

16. ₹ R की स्थायी वार्षिकी, जो कि प्रत्येक भुगतान अवधि के अंत में अदा करनी होती है, का वर्तमान मूल्य, जबकि पैसा प्रति अवधि, i के लायक है, है :

(a) Ri (b) $R + \frac{R}{i}$

(c) $\frac{R}{i}$ (d) R – Ri

17. वह प्रभावी दर जो 10% वार्षिक, प्रति तिमाही संकलित होने वाली अंकित दर के तुल्य है, है :

(a) 10·25% (b) 10·38%

(c) 10·47% (d) 10·53%

18. $x \geq 0, y \geq 0$ द्वारा निर्धारित क्षेत्र स्थित है :

(a) I चतुर्थांश में (b) II चतुर्थांश में

(c) III चतुर्थांश में (d) IV चतुर्थांश में

*प्रश्न संख्या **19** और **20** अभिकथन एवं तर्क आधारित प्रश्न हैं और प्रत्येक प्रश्न का 1 अंक है । दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है । इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (a), (b), (c) और (d) में से चुनकर दीजिए ।*

(a) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है ।

(b) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है ।

(c) अभिकथन (A) सही है तथा तर्क (R) ग़लत है ।

(d) अभिकथन (A) ग़लत है तथा तर्क (R) सही है ।

19. *अभिकथन (A) :* फलन $f(x) = (x + 2)\, e^{-x}$, अंतराल $(-1, \infty)$ में वर्धमान है ।

तर्क (R) : एक फलन f(x) वर्धमान है, यदि $f'(x) > 0$ है ।

16. The present value of a perpetuity of ₹ R payable at the end of each payment period, when the money is worth i per period, is given by :

(a) Ri (b) $R + \frac{R}{i}$

(c) $\frac{R}{i}$ (d) $R - Ri$

17. The effective rate which is equivalent to nominal rate of 10% p.a. compounded quarterly is :

(a) 10·25% (b) 10·38%

(c) 10·47% (d) 10·53%

18. Region represented by $x \geq 0$, $y \geq 0$ lies in

(a) I quadrant (b) II quadrant

(c) III quadrant (d) IV quadrant

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* The function $f(x) = (x + 2)\, e^{-x}$ is increasing in the interval $(-1, \infty)$.

Reason (R) : A function f(x) is increasing, if $f'(x) > 0$.

20. *अभिकथन (A) :* परवलय $y^2 = 4ax$ के कुल को निरूपित करने वाला अवकल समीकरण $x\frac{dy}{dx} - 2y = 0$ है, जबकि 'a' एक प्राचल है ।

तर्क (R) : यदि दिए गए वक्रों के कुल में 'n' प्राचल हों, तो इसे n बार अवकलित किया जाता है ताकि प्राचल को लुप्त कर n^{th} कोटि का अवकल समीकरण प्राप्त किया जा सके ।

खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं ।

21. (क) दो पाइप A तथा B एक टैंक को क्रमश: 24 मिनट तथा 32 मिनट में भर सकती हैं । यदि दोनों पाइपों को एक साथ खोल दिया जाए तो पाइप B को कितने समय के बाद बन्द कर देना चाहिए ताकि टैंक 18 मिनट में भर जाए ?

अथवा

(ख) एक-किलोमीटर की एक दौड़ में, A 30 सेकंड से B को हराता है तथा B 15 सेकंड से C को हराता है । यदि A, C को 180 मी. से हराता है, तो A द्वारा एक किलोमीटर दौड़ने में लिया गया समय ज्ञात कीजिए ।

22. x के लिए हल कीजिए : $\frac{x+3}{x-2} \leq 2.$

23. (क) निम्न रैखिक समीकरण निकाय को क्रैमर नियम से हल कीजिए :

$$2x - y = 17, \quad 3x + 5y = 6$$

अथवा

(ख) x के वह पूर्णांक मान ज्ञात कीजिए जिनके लिए आव्यूह $A = \begin{bmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{bmatrix}$ अव्युत्क्रमणीय है ।

20. *Assertion (A) :* The differential equation representing the family of parabolas $y^2 = 4ax$, where 'a' is a parameter, is $x\frac{dy}{dx} - 2y = 0$.

Reason (R) : If the given family of curves has n parameters, then it is to be differentiated n times to eliminate the parameter and obtain the n^{th} order differential equation.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Two pipes A and B can fill a tank in 24 minutes and 32 minutes respectively. If both the pipes are opened simultaneously, after how much time should B be closed so that the tank is full in 18 minutes ?

OR

(b) In a one-kilometre race, A beats B by 30 seconds and B beats C by 15 seconds. If A beats C by 180 metres, then find the time taken by A to run 1 kilometre.

22. Solve for x : $\frac{x+3}{x-2} \leq 2$.

23. (a) Solve the following system of equations by Cramer's rule :

$$2x - y = 17, \quad 3x + 5y = 6$$

OR

(b) Determine the integral value(s) of x for which the matrix A is singular :

$$A = \begin{bmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{bmatrix}$$

24. एक कण वक्र $6y = x^3 + 2$ की दिशा में चल रहा है । वक्र पर वह बिंदु ज्ञात कीजिए जिन पर y-निर्देशांक के परिवर्तन की दर, x-निर्देशांक के परिवर्तन की दर की 8 गुनी है ।

25. माना कि एक फैक्टरी द्वारा उत्पादित वस्तुओं में से 2% खराब हैं । प्रायिकता ज्ञात कीजिए कि यादृच्छया लिए गए 100 वस्तुओं के एक नमूने में 3 खराब वस्तुएँ हैं । (दिया है $e^{-2} = 0{\cdot}135$)

खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं ।

26. (क) डिट्टोल से भरी हुई एक बोतल ली गई तथा इसमें से एक-तिहाई डिट्टोल निकाल कर उतना ही पानी डाल कर बोतल को फिर से भर दिया । ऐसा तीन बार किया गया । अन्त में बोतल में डिट्टोल की मात्रा से पानी की मात्रा का अनुपात ज्ञात कीजिए ।

अथवा

(ख) एक पाइप A एक टैंक को 3 घंटे में भर सकती है । इस टैंक के साथ दो निकासी पाइप B तथा C लगे हैं जो कि टैंक को क्रमश: 7 घंटे तथा 10 घंटे में खाली कर सकते हैं । यदि तीनों पाइप एक साथ खोल दिए जाएँ, तो टैंक को भरने में कितना समय लगेगा ?

27. फलन $f(x) = x^3 - 6x^2 + 9x - 8$ के सभी स्थानीय उच्चतम या स्थानीय निम्नतम बिंदु ज्ञात कीजिए ।

28. एक अनभिनत पासे को बार-बार तब तक उछाला गया जब तक कि इस पर तीसरी बार छ: नहीं आ जाता । इस पासे को छठी बार उछालने पर तीसरी बार छ: आने की प्रायिकता ज्ञात कीजिए ।

29. एक चार-पाहिया वाहन की साप्ताहिक माध्य बिक्री, 20 एजेंसियों में 50 इकाई प्रति एजेंसी थी । विज्ञापन द्वारा प्रचार करने पर, साप्ताहिक माध्य बिक्री प्रति एजेंसी बढ़कर 55 इकाई हो गई जबकि मानक विचलन 10 इकाई था । जाँच कीजिए कि क्या यह विज्ञापन प्रचार सफल था ।

(प्रयोग कीजिए $t_{0{\cdot}005} = 1{\cdot}729$, 19 d.f. के लिए)

24. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the ordinate is changing 8 times as fast as abscissa.

25. Suppose 2% of the items made by a factory are defective. Find the probability that there are 3 defective items in a sample of 100 items selected at random. (Given $e^{-2} = 0{\cdot}135$)

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) A bottle is full of dettol. One-third of its dettol is taken away and an equal amount of water is poured into the bottle to fill it again. This operation is repeated three times. Find the final ratio of dettol to water in the bottle.

OR

(b) A pipe A can fill a tank in 3 hours. There are two outlet pipes B and C from the tank which can empty it in 7 and 10 hours respectively. It all the three pipes are opened simultaneously, how long will it take to fill the tank ?

27. Find all the points of local maxima and local minima for the function $f(x) = x^3 - 6x^2 + 9x - 8$.

28. An unbiased die is thrown again and again until three sixes are obtained. Find the probability of obtaining a third six in the sixth throw of the die.

29. The mean weekly sales of a four-wheeler were 50 units per agency in 20 agencies. After an advertising campaign, the mean weekly sales increased to 55 units per agency with standard deviation of 10 units. Test whether the advertising campaign was successful.
(Use $t_{0{\cdot}005} = 1{\cdot}729$ for 19 d.f.)

30. (क) एक संपत्ति का मूल्य ₹ 4,50,000 है जबकि इसकी अनुमानित आयु 5 वर्ष है तथा इसका अवशिष्ट मूल्य ₹ 1,00,000 है । रैखिक अवमूल्यन विधि द्वारा इस संपत्ति का वार्षिक अवमूल्यन ज्ञात कीजिए तथा एक वार्षिक अवमूल्यन अनुसूची बनाइए ।

अथवा

(ख) अमृता ने ₹ 12,50,000 मूल्य की एक कार खरीदी, जिसके लिए उसने ₹ 3,00,000 तत्काल अदायगी की तथा शेष राशि को 4 वर्षों में समान मासिक किश्तों में 15% वार्षिक ब्याज की दर पर वापिस करना है । ज्ञात कीजिए कि अमृता को कितनी ई.एम.आई. (EMI) देनी होगी । {दिया है $(1{\cdot}0125)^{-48} = 0{\cdot}5508565$)}

31. व्यवरोधों

$$x + y \leq 24,$$

$$2x + y \leq 32,$$

$x \geq 0, y \geq 0$ के अंतर्गत,

$z = 300x + 190y$ का अधिकतमीकरण कीजिए ।

खण्ड घ

इस खण्ड में दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं ।

32. (क) आव्यूह

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

का व्युत्क्रम (A^{-1}) ज्ञात कीजिए, अत: दर्शाइए कि $AA^{-1} = I$.

अथवा

30. (a) An asset costs ₹ 4,50,000 with an estimated useful life of 5 years and a scrap value of ₹ 1,00,000. Using linear depreciation method, find the annual depreciation of the asset and construct a yearly depreciation schedule.

OR

(b) Amrita bought a car worth ₹ 12,50,000 and makes a down payment of ₹ 3,00,000. The balance amount is to be paid in 4 years by equal monthly instalments at an interest rate of 15% p.a. Find the EMI that Amrita has to pay for the car.

{Given $(1{\cdot}0125)^{-48} = 0{\cdot}5508565$)}

31. Maximise $z = 300x + 190y$

subject to constraints :

$x + y \leq 24$,

$2x + y \leq 32$,

$x \geq 0, y \geq 0$.

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Find the inverse of the matrix :

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

and hence show that $AA^{-1} = I$.

OR

(ख) आव्यूह विधि के प्रयोग से निम्न रैखिक समीकरण निकाय का x, y, z के लिए हल ज्ञात कीजिए :

$$x - y + z = 4$$
$$2x + y - 3z = 0$$
$$x + y + z = 2$$

33. (क) संख्या 15 को ऐसे दो भागों में बाँटें कि पहले भाग के वर्ग तथा दूसरे भाग के घन का गुणनफल अधिकतम हो ।

अथवा

(ख) वक्र $y^2 = 2x$ पर वह बिंदु ज्ञात कीजिए जो कि बिंदु (1, 4) से निकटतम हो ।

34. निम्न आँकड़ों के लिए न्यूनतम वर्गों की विधि से एक सरल-रेखीय उपनति फिट कीजिए तथा उपनति मान ज्ञात कीजिए :

वर्ष :	2010	2012	2013	2014	2015	2016	2019
बिक्री (लाख ₹ में) :	65	68	70	72	75	67	73

35. चक्रवृद्धि वार्षिक वृद्धि दर (CAGR) को परिभाषित कीजिए तथा इसे ज्ञात करने का सूत्र दीजिए । इस सूत्र के प्रयोग से विकास के निवेश का CAGR ज्ञात कीजिए जबकि दिया है कि : विकास ने एक कम्पनी के स्टॉक में 6 वर्ष के लिए ₹ 10,000 का निवेश किया । प्रत्येक वर्ष के अंत में उसके निवेश का मान निम्न है :

वर्ष 1	वर्ष 2	वर्ष 3	वर्ष 4	वर्ष 5	वर्ष 6
₹ 11,000	₹ 11,500	₹ 11,650	₹ 11,800	₹ 12,200	₹ 14,000

$[(1{\cdot}4)^{1/6} = 1{\cdot}058$ का प्रयोग कीजिए]

(b) Using matrix method, solve the following system of equations for x, y and z :

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

33. (a) Divide a number 15 into two parts such that the square of one part multiplied with the cube of the other part is maximum.

OR

(b) Find a point on the curve $y^2 = 2x$ which is nearest to the point (1, 4).

34. Fit a straight line trend by method of least squares to the following data and find the trend values :

Year :	2010	2012	2013	2014	2015	2016	2019
Sales (in lakh ₹) :	65	68	70	72	75	67	73

35. Define Compound Annual Growth Rate (CAGR) and give the formula for calculating CAGR. Using the formula, calculate CAGR of Vikas's investment given below :

Vikas invested ₹ 10,000 in a stock of a company for 6 years. The value of his investment at the end of each year is given below :

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
₹ 11,000	₹ 11,500	₹ 11,650	₹ 11,800	₹ 12,200	₹ 14,000

[Use $(1{\cdot}4)^{1/6} = 1{\cdot}058$]

खण्ड ङ

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं जिनमें प्रत्येक के 4 अंक हैं ।

प्रकरण अध्ययन – 1

36. एक फैक्ट्री बल्ब बनाती है । बल्बों की एक बहुत बड़ी ढेरी में 6% बल्ब खराब होते हैं ।

उपर्युक्त सूचनाओं के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) प्रायिकता ज्ञात कीजिए कि यादृच्छया चुने गए 100 बल्बों के एक नमूने में कोई भी खराब बल्ब नहीं है । [$e^{-6} = 0{\cdot}0024$ लीजिए] *1*

(ii) प्रायिकता ज्ञात कीजिए कि 100 बल्बों के नमूने में मात्र 2 खराब बल्ब हैं । *1*

(iii) (क) प्रायिकता ज्ञात कीजिए कि 100 बल्बों के नमूने में एक से अधिक खराब बल्ब नहीं है । *2*

अथवा

(iii) (ख) 100 बल्बों के नमूने में खराब बल्ब की प्रायिकता बंटन का माध्य तथा प्रसरण ज्ञात कीजिए । *2*

प्रकरण अध्ययन – 2

37. एक फैक्ट्री में टैनिस के रैकिट तथा क्रिकेट के बैट बनते हैं । एक टैनिस रैकिट बनाने में $1\frac{1}{2}$ घंटे मशीन का समय तथा 3 घंटे का समय शिल्पकारिता में लगता है, जबकि एक क्रिकेट बैट बनाने में 3 घंटे मशीन का समय तथा 1 घंटे का समय शिल्पकारिता में लगता है । एक दिन में फैक्ट्री के पास मशीन का समय 42 घंटे से अधिक नहीं है जबकि शिल्पकारिता का समय 24 घंटे से अधिक नहीं है । एक रैकिट तथा एक बैट पर लाभ क्रमश: ₹ 20 तथा ₹ 10 है ।

उपर्युक्त सूचनाओं के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) यदि फैक्ट्री द्वारा बनाए गए बैट तथा रैकिटों की संख्या क्रमश: x तथा y है, तो कुल लाभ का व्यंजक लिखिए । *1*

(ii) शिल्पकारिता पर लगे घंटों की संख्या से संबंधित व्यवरोध लिखिए । *1*

(iii) (क) फैक्ट्री द्वारा (₹ में) अर्जित अधिकतम लाभ ज्ञात कीजिए । *2*

अथवा

(iii) (ख) अधिकतम लाभ के लिए कितने बैट तथा रैकिट, क्रमश: बनाए जाने चाहिए ? *2*

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A factory produces bulbs, of which 6% are defective bulbs in a large bulk of bulbs.

Based on the above information, answer the following questions :

(i) Find the probability that in a sample of 100 bulbs selected at random, none of the bulbs is defective. (Use : $e^{-6} = 0{\cdot}0024$) *1*

(ii) Find the probability that the sample of 100 bulbs has exactly two defective bulbs. *1*

(iii) (a) Find the probability that the sample of 100 bulbs will include not more than one defective bulb. *2*

OR

(iii) (b) Find the mean and the variance of the distribution of number of defective bulbs in a sample of 100 bulbs. *2*

Case Study – 2

37. A factory manufactures tennis rackets and cricket bats. A tennis racket takes $1\frac{1}{2}$ hours of machine time and 3 hours of craftsmanship in its making; while a cricket bat takes 3 hours of machine time and 1 hour of craftsmanship. In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsmanship. Profit on a racket and on a bat are ₹ 20 and ₹ 10 respectively.

Based on the above information, answer the following questions :

(i) If x and y are the numbers of bats and rackets manufactured by the factory, then write the expression of total profit. *1*

(ii) Write the constraint that relates the number of craftsmanship hours. *1*

(iii) (a) Determine the maximum profit (in ₹) earned by the factory. *2*

OR

(iii) (b) How many bats and rackets respectively, are to be manufactured to earn maximum profit ? *2*

प्रकरण अध्ययन – 3

38. वर्ष 2010 में, श्री अग्रवाल ने स्टेट बैंक ऑफ इंडिया से 20 वर्ष के लिए, ₹ 30,00,000 का गृह ऋण 7·5% वार्षिक दर पर लिया गया जबकि ब्याज मासिक संयोजित होता है ।

उपर्युक्त सूचनाओं के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) ई.एम.आई. (EMI) ज्ञात कीजिए । *1*

(ii) श्री अग्रवाल द्वारा 150वीं किश्त में अदा की गई मूलधन की राशि ज्ञात कीजिए । *1*

(iii) (क) श्री अग्रवाल द्वारा दी गई कुल ब्याज की राशि ज्ञात कीजिए । *2*

अथवा

(iii) (ख) श्री अग्रवाल द्वारा पूरे गृह ऋण की अदायगी में कुल कितनी राशि दी गई ? *2*

$[(1{\cdot}00625)^{240} = 4{\cdot}4608; (1{\cdot}00625)^{91} = 1{\cdot}7629$ लीजिए]

Case Study – 3

38. In the year 2010, Mr. Aggarwal took a home loan of ₹ 30,00,000 from State Bank of India at 7·5% p.a. compounded monthly for 20 years.

Based on the above information, answer the following questions :

(i) Determine the EMI. *1*

(ii) Find the principal paid by Mr. Aggarwal in the 150^{th} instalment. *1*

(iii) (a) Find the total interest paid by Mr. Aggarwal. *2*

OR

(iii) (b) How much was paid by Mr. Aggarwal to repay the entire amount of home loan ? *2*

[Use $(1{\cdot}00625)^{240} = 4{\cdot}4608$; $(1{\cdot}00625)^{91} = 1{\cdot}7629$]

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior School Certificate Examination, 2023
SUBJECT NAME: APPLIED MATHEMATICS (SUBJECT CODE S46547A)
(PAPER CODE 465)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-X, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after delibration and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark(√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right **(✓)**while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.

8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note **"Extra Question"**.
10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks _____(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
15	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

Set 4

M.S. Code **S46547A**

MARKING SCHEME

APPLIED MATHEMATICS

Section A

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of **1 mark each.**	
1.	The last (unit) digit of $(22)^{12}$ is : (a) 2 (b) 4 (c) 6 (d) 8	
Sol.	(c) 6	**(1)**
2.	The least non-negative remainder, when 3^{15} is divided by 7 is : (a) 1 (b) 5 (c) 6 (d) 7	
Sol.	(c) 6	**(1)**
3.	If $A=\begin{bmatrix}1 & 0\\ -2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-5 & 10\\ -10 & -5\end{bmatrix}$, then AB is : (a) $\begin{bmatrix}-5 & 10\\ 0 & -5\end{bmatrix}$ (b) $\begin{bmatrix}0 & -5\\ 25 & 10\end{bmatrix}$ (c) $\begin{bmatrix}10 & -25\\ -5 & 0\end{bmatrix}$ (d) $\begin{bmatrix}-5 & 10\\ 0 & -25\end{bmatrix}$	

Sol.	(d) $\begin{bmatrix} -5 & 10 \\ 0 & -25 \end{bmatrix}$	**(1)**
4.	If $\begin{bmatrix} x+y & x+2 \\ 2x-y & 16 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 1 & 3y+1 \end{bmatrix}$, then the values of x and y are : (a) $x = 3, y = 5$ (b) $x = 5, y = 3$ (c) $x = 2, y = 7$ (d) $x = 7, y = 2$	
Sol.	(a) $\boldsymbol{x = 3, y = 5}$	**(1)**
5.	The ratio in which a grocer mixes two varieties of pulses costing ₹ 85 per kg and ₹ 100 per kg respectively so as to get a mixture worth ₹ 92 per kg, is : (a) 7 : 8 (b) 8 : 7 (c) 5 : 7 (d) 7 : 5	
Sol.	(b) 8 : 7	**(1)**
6.	If $\frac{\lvert x+1 \rvert}{x+1} > 0, x \in \mathbb{R}$, then : (a) $x \in [-1, \infty)$ (b) $x \in (-1, \infty)$ (c) $x \in (-\infty, -1)$ (d) $x \in (-\infty, -1]$	
Sol.	(b) $\boldsymbol{x \in (-1, \infty)}$	**(1)**
7.	A and B are square matrices each of order 3 such that $\lvert A \rvert = -1$ and $\lvert B \rvert = 3$. What is the value of $\lvert 3AB \rvert$? (a) – 9 (b) – 18 (c) – 27 (d) – 81	
Sol.	(d) $\boldsymbol{-81}$	**(1)**

8.	If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is : (a) -1 (b) 0 (c) 1 (d) 3	
Sol.	(a) $\mathbf{-1}$	**(1)**
9.	The relation between 'Marginal cost' and 'Average cost' of producing 'x' units of a product is : (a) $\frac{d(AC)}{dx} = x(MC - AC)$ (b) $\frac{d(AC)}{dx} = x(AC - MC)$ (c) $\frac{d(AC)}{dx} = \frac{1}{x}(AC - MC)$ (d) $\frac{d(AC)}{dx} = \frac{1}{x}(MC - AC)$	
Sol.	(d) $\boldsymbol{\frac{d(AC)}{dx} = \frac{1}{x}(MC - AC)}$	**(1)**
10.	$\int (x-1)e^{-x}\,dx$ is equal to : (a) $(x-2)e^{-x} + C$ (b) $xe^{-x} + C$ (c) $-xe^{-x} + C$ (d) $(x+1)e^{-x} + C$	
Sol.	(c) $\mathbf{-xe^{-x} + C}$	**(1)**
11.	The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is : (a) $\frac{1}{x} + \frac{1}{y} = C$ (b) $xy = C$ (c) $\log x \log y = C$ (d) $x + y = C$	
Sol.	(b) $\boldsymbol{xy = C}$	**(1)**

12.	If X is a Poisson variable such that P(X = 1) = 2P(X = 2), then P(X = 0) is : (a) e (b) $\frac{1}{e}$ (c) 1 (d) e^2	
Sol.	(b) $\frac{1}{e}$	**(1)**
13.	If the calculated value of $\|t\| < t_v(\alpha)$, then the null hypothesis is : (a) rejected (b) accepted (c) cannot be determined (d) neither accepted nor rejected	
Sol.	(b) accepted	**(1)**
14.	For testing the significance of difference between the means of two independent samples, the degree of freedom (v) is taken as : (a) $n_1 - n_2 + 2$ (b) $n_1 - n_2 - 2$ (c) $n_1 + n_2 - 2$ (d) $n_1 + n_2 - 1$	
Sol.	(c) $\boldsymbol{n_1 + n_2 - 2}$	**(1)**
15.	The straight line trend is represented by the equation : (a) $y_c = a + bx$ (b) $y_c = a - bx$ (c) $y_c = na + b\Sigma x$ (d) $y_c = na - b\Sigma x$	
Sol.	(a) $\boldsymbol{y_c = a + bx}$	**(1)**

16.	The present value of a perpetuity of ₹ R payable at the end of each payment period, when the money is worth i per period, is given by : (a) Ri (b) $R + \frac{R}{i}$ (c) $\frac{R}{i}$ (d) $R - Ri$	
Sol.	(c) $\frac{R}{i}$	**(1)**
17.	The effective rate which is equivalent to nominal rate of 10% p.a. compounded quarterly is : (a) 10·25% (b) 10·38% (c) 10·47% (d) 10·53%	
Sol.	(b) 10.38%	**(1)**
18.	Region represented by $x \geq 0$, $y \geq 0$ lies in (a) I quadrant (b) II quadrant (c) III quadrant (d) IV quadrant	
Sol.	(a) I quadrant	**(1)**
	*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.* (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (c) Assertion (A) is true and Reason (R) is false. (d) Assertion (A) is false and Reason (R) is true.	
19.	*Assertion (A) :* The function $f(x) = (x + 2)\, e^{-x}$ is increasing in the interval $(-1, \infty)$. *Reason (R) :* A function f(x) is increasing, if $f'(x) > 0$.	
Sol.	(d) Assertion (A) is false and Reason (R) is true.	**(1)**

20.	*Assertion (A) :* The differential equation representing the family of parabolas $y^2 = 4ax$, where 'a' is a parameter, is $x\frac{dy}{dx} - 2y = 0$. *Reason (R) :* If the given family of curves has n parameters, then it is to be differentiated n times to eliminate the parameter and obtain the n^{th} order differential equation.	
Sol.	(d) Assertion (A) is false and Reason (R) is true.	**(1)**
	SECTION B This section comprises very short answer (VSA) type questions of **2 marks each.**	
21(a).	Two pipes A and B can fill a tank in 24 minutes and 32 minutes respectively. If both the pipes are opened simultaneously, after how much time should B be closed so that the tank is full in 18 minutes ?	
Sol.	Let B be closed after n minutes. Then, pipe A runs for 18 minutes and B runs for n minutes to fill the tank. $\therefore \frac{18}{24} + \frac{n}{32} = 1$ $\Rightarrow \frac{3}{4} + \frac{n}{32} = 1 \Rightarrow n = 8.$ Hence, pipe B must be closed after 8 min	**(1)** **(1)**
	OR	
21(b).	In a one-kilometre race, A beats B by 30 seconds and B beats C by 15 seconds. If A beats C by 180 metres, then find the time taken by A to run 1 kilometre.	
Sol.	Suppose A takes 't' seconds to run 1 km race. Then, B takes (t + 30) seconds and C takes (t + 30 + 15) seconds, i.e. (t + 45) seconds. We find A beats C by (30 + 15) seconds = 45 seconds and it is given that A beats C by 180 metres.	

	$\therefore$ C runs 180 m in 45 seconds $\Rightarrow$ C runs 1000 m in $\left(\frac{45}{180} \times 1000\right)$ seconds = 250 seconds. $\therefore$ t + 45 = 250 $\Rightarrow$ t = 205 Hence, A takes 205 seconds to run 1 km	$(\frac{1}{2})$ **(1)** $(\frac{1}{2})$
22.	Solve for x : $\frac{x+3}{x-2} \leq 2$.	
Sol.	$\frac{x+3}{x-2} - 2 \leq 0 \Rightarrow \frac{-x+7}{x-2} \leq 0$ or $\frac{x-7}{x-2} \geq 0$ Thus, the solution set is $(-\infty, 2) \cup [7, \infty)$	**(1)** **(1)**
23(a).	Solve the following system of equations by Cramer's rule : $2x - y = 17,\ 3x + 5y = 6$	
Sol.	Here, $D = \begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix} = 13$ $D_1 = \begin{vmatrix} 17 & -1 \\ 6 & 5 \end{vmatrix} = 91$ $D_2 = \begin{vmatrix} 2 & 17 \\ 3 & 6 \end{vmatrix} = -39$ Thus, $x = \frac{D_1}{D} = 7$; $y = \frac{D_2}{D} = -3$	$(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$
	OR	
23(b).	Determine the integral value(s) of x for which the matrix A is singular : $A = \begin{bmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{bmatrix}$	
Sol.	A is singular gives $\begin{vmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{vmatrix} = 0$ i.e. $(x + 1)[(x + 2)(x - 6) - 2] + 3[-5x + 30 - 8] + 4[-5 - 4x - 8] = 0$	$(\frac{1}{2})$

	i.e. $(x + 1)\ (x^2 - 4x - 14) - 15x + 66 - 52 - 16x = 0$ i.e. $x^3 - 3x^2 - 49x = 0$ $x = 0, \frac{3 \pm \sqrt{205}}{2}$ Hence, $x = 0$ is the only integral value.	**(1)** $(\frac{1}{2})$
24.	A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the ordinate is changing 8 times as fast as abscissa.	
Sol.	Here, $6y = x^3 + 2$ $\Rightarrow 6\frac{dy}{dt} = 3x^2\frac{dx}{dt}$ As $\frac{dy}{dt} = 8\frac{dx}{dt}$, we have $48\frac{dx}{dt} = 3x^2\frac{dx}{dt} \Rightarrow x = 4, -4$ when $x = 4$, $y = 11$; when $x = -4$, $y = \frac{-31}{3}$. $\therefore$ Points on the curve are $(4, 11), \left(-4, \frac{-31}{3}\right)$	$(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$
25.	Suppose 2% of the items made by a factory are defective. Find the probability that there are 3 defective items in a sample of 100 items selected at random. (Given $e^{-2} = 0{\cdot}135$)	
Sol.	Let p be the probability that an item is defective so, $p = \frac{2}{100} = 0{\cdot}02$. Here $n = 100\ \therefore m = np = 2$ $P(X = r) = \frac{m^r}{r!}e^{-m} = \frac{2^r e^{-2}}{r!}$ $\Rightarrow \mathbf{P(X = 3) = \frac{2^3 e^{-2}}{3!} = \frac{4}{3} \times 0{\cdot}135 = 0{\cdot}18}$	$(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$ $(\frac{1}{2})$
	SECTION C	

	This section comprises short answer (SA) type questions of **3 marks each.**	
26(a).	A bottle is full of dettol. One-third of its dettol is taken away and an equal amount of water is poured into the bottle to fill it again. This operation is repeated three times. Find the final ratio of dettol to water in the bottle.	
Sol.	Let the original quantity of dettol be x litres and the quantity of Dettol replaced by water be y litres. So, $y = \frac{x}{3}$. After 3 operations the quantity of dettol left = $x\left(1 - \frac{y}{x}\right)^3$. **(1)** After 3 operations the quantity of water in the bottle = $x - x\left(1 - \frac{x}{3x}\right)^3$ **(1)** Hence, the required ratio is $x\left(1 - \frac{x}{3x}\right)^3 : [x - x\left(1 - \frac{x}{3x}\right)^3]$ $= \left(1 - \frac{1}{3}\right)^3 : [1 - \left(1 - \frac{1}{3}\right)^3]$ $= \frac{8}{27} : \frac{19}{27}$ $= 8 : 19$ **(1)**	
26(b).	A pipe A can fill a tank in 3 hours. There are two outlet pipes B and C from the tank which can empty it in 7 and 10 hours respectively. It all the three pipes are opened simultaneously, how long will it take to fill the tank ?	
Sol.	Here, $n_A = 3$, $n_B = 7$ and $n_C = 10$. $\frac{1}{n} = \frac{1}{n_A} - \frac{1}{n_B} - \frac{1}{n_C}$ $\Rightarrow \frac{1}{n} = \frac{1}{3} - \frac{1}{7} - \frac{1}{10}$ **(2)** $\Rightarrow \frac{1}{n} = \frac{19}{210} \Rightarrow n = 11\frac{1}{19}$ **(1)** Hence, the tank is filled in $11\frac{1}{19}$ hours.	

27.	Find all the points of local maxima and local minima for the function $f(x) = x^3 - 6x^2 + 9x - 8$.	
Sol.	$y = x^3 - 6x^2 + 9x - 8$ $\Rightarrow \frac{dy}{dx} = 3x^2 - 12x + 9$ $\Rightarrow \frac{dy}{dx} = 3(x-1)(x-3)$ Critical points are 1, 3 Showing, x=1 is a point of local maxima. Showing, x=3 is a point of local minima.	**(1)** **(1)** $(\frac{1}{2})$ $(\frac{1}{2})$
28.	An unbiased die is thrown again and again until three sixes are obtained. Find the probability of obtaining a third six in the sixth throw of the die.	
Sol.	Let A be the event of obtaining two sixes in the first five throws of a die. Let B be the event of obtaining a six in the sixth throw of a die. Then required probability = P(AB) = P(A) P(B) Here, $P(B) = \frac{1}{6}$ and $P(A) = 5_{C_2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3 = \frac{625}{3888}$ Thus, Required probability = $\frac{625}{3888} \times \frac{1}{6} = \frac{625}{23328}$	**(2)** **(1)**
29.	The mean weekly sales of a four-wheeler were 50 units per agency in 20 agencies. After an advertising campaign, the mean weekly sales increased to 55 units per agency with standard deviation of 10 units. Test whether the advertising campaign was successful. (Use $t_{0\cdot005} = 1\cdot729$ for 19 d.f.)	

Sol. We are given

$\mu = 50,\ \bar{x} = 55, SD = 10, n = 20$ **(1)**

$H_0: \mu = 50$

$H_1: \mu > 50$

$t = \frac{\bar{x} - \mu}{\frac{SD}{\sqrt{n}}} = \frac{55 - 50}{\frac{10}{\sqrt{20}}} = 2{\cdot}236$ **(2)**

$t_{cal\ value} > t_{tab\ value}$

Hence H_0 is rejected.

So, Advertising Campaign was successful.

30(a). An asset costs ₹ 4,50,000 with an estimated useful life of 5 years and a scrap value of ₹ 1,00,000. Using linear depreciation method, find the annual depreciation of the asset and construct a yearly depreciation schedule.

Sol. Here C = ₹ 4,50,000 **(2)**

S = ₹ 1,00,000

and n = 5 years.

Annual depreciation $D = \frac{C - S}{n} = ₹\ 70,000$

Thus, yearly depreciation schedule is as follows:

Years	Book value at the beginning of the year (in ₹)	Depreciation (in ₹)	Book value at the end of the year (in ₹)
1	4,50,000	70,000	3,80,000
2	3,80,000	70,000	3,10,000

	3	3,10,000	70,000	2,40,000	**(1 for correct table)**
	4	2,40,000	70,000	1,70,000	
	5	1,70,000	70,000	1,00,000	

30(b). Amrita bought a car worth ₹ 12,50,000 and makes a down payment of ₹ 3,00,000. The balance amount is to be paid in 4 years by equal monthly instalments at an interest rate of 15% p.a. Find the EMI that Amrita has to pay for the car.

{Given $(1{\cdot}0125)^{-48} = 0{\cdot}5508565$)}

Sol. Here P = ₹ 9,50,000, i = $\frac{15}{1200}$ = 0·0125 **(½)**

n = 48 months **(½)**

Using the reducing balancing method, **(1)**

$$E = \frac{Pi}{1 - (1+i)^{-n}} = \frac{9,5,0000 \times 0{\cdot}0125}{1 - (1 + 0{\cdot}0125)^{-48}}$$ **(½)**

$$= \frac{11875}{1 - (1.0125)^{-48}} = \frac{11875}{1 - 0.5508565}$$ **(½)**

= ₹ 26,439·21

31. Maximise z = 300x + 190y

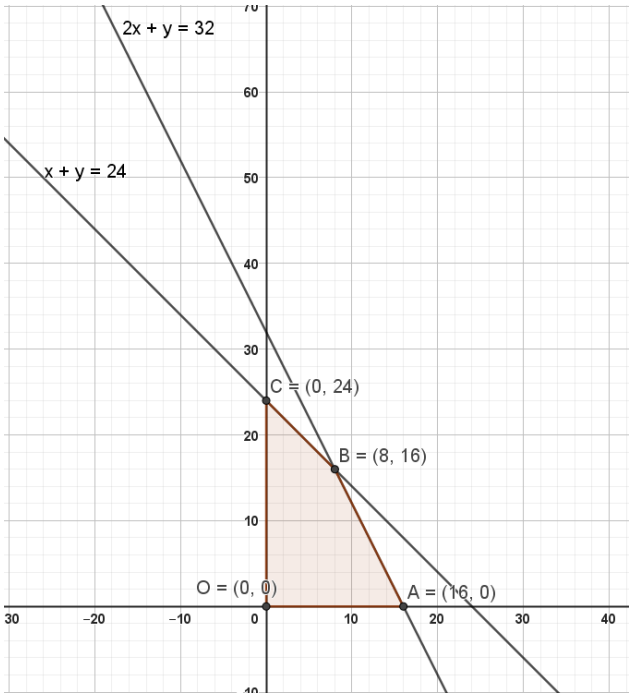
subject to constraints :

$x + y \leq 24$,

$2x + y \leq 32$,

$x \geq 0, y \geq 0$.

Sol.



(2)

Corner Points	Value of Z
O (0,0)	0
A (16,0)	4800
B (8,16)	5440 → Max Value
C (0,24)	4560

(1)

So Z is maximum at B (8,16)

Max Value of Z = 5440

SECTION D

This section comprises of Long Answer (LA) type questions of 5 marks each.

32(a).	Find the inverse of the matrix : $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ and hence show that $AA^{-1} = I$.	
Sol.	Here, $\lvert A\rvert = -(-4-3) - (12+1) + 2(9-1)$ $= 7 - 13 + 16 = 10 \neq 0$ $\Rightarrow \text{adj}(A) = \begin{bmatrix} -7 & -13 & 8 \\ 2 & -2 & 2 \\ 3 & 7 & -2 \end{bmatrix}^T = \begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix}$ Hence $A^{-1} = \frac{1}{10}\begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix}$ $AA^{-1} = \frac{1}{10}\begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}\begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**(1)** **$(2\frac{1}{2})$** **$(\frac{1}{2})$** **(1)**
	OR	
32(b).	Using matrix method, solve the following system of equations for x, y and z : $x - y + z = 4$ $2x + y - 3z = 0$ $x + y + z = 2$	
Sol.	The matrix equation AX = B is $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$	**$(\frac{1}{2})$**

	$\|A\| = 10$ adj A $= \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}' = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ Here $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ So, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ Thus, x = 2, y = – 1, z = 1	**(1)** **(2)** **($\frac{1}{2}$)** **(1)**
33(a).	Divide a number 15 into two parts such that the square of one part multiplied with the cube of the other part is maximum.	
Sol.	Let the two parts be x and 15 – x. Then, let $y = x^2(15 - x)^3$ $\Rightarrow \frac{dy}{dx} = x(15 - x)^2 (-5x + 30)$ $\frac{dy}{dx} = 0$ gives x = 0, 15, 6 Rejecting x = 0, 15. Hence x = 6 Showing, $x = 6$ is a point of maxima So, y is maximum when x = 6. Hence two parts are 6 and 9	**(1)** **(1)** **($1\frac{1}{2}$)** **(1)** **($\frac{1}{2}$)**
	OR	
33(b).	Find a point on the curve $y^2 = 2x$ which is nearest to the point (1, 4).	

Sol.

Let P (x, y) be the required point which is nearest to Q (1, 4). Then distance PQ should be minimum and hence $(PQ)^2$ should be minimum. $(\frac{1}{2})$

Now, $(PQ)^2 = (x-1)^2 + (y-4)^2 = \left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2$ **(1)**

$= \frac{y^4 - 32y + 68}{4}$ **(1)**

Let $D = \frac{y^4 - 32y + 68}{4}$

$\frac{dD}{dy} = y^3 - 8$ $(\frac{1}{2})$

$\frac{dD}{dy} = 0 \Rightarrow y = 2$ **(1)**

Showing, y = 2 is a point of minima $(\frac{1}{2})$

Thus, the point is (2, 2) $(\frac{1}{2})$

34. Fit a straight line trend by method of least squares to the following data and find the trend values :

Year :	2010	2012	2013	2014	2015	2016	2019
Sales (in lakh ₹) :	65	68	70	72	75	67	73

Sol. Consider year 2014 as the year of origin. Calculation of trend values by method of least squares.

Year	Sales (in lakh ₹) y	Deviations from 2014 (x)	Squares of Deviations (x^2)	Sales deviation (xy)
2010	65	– 4	16	– 260
2012	68	– 2	4	– 136
2013	70	– 1	1	– 70

2014	72	0	0	0
2015	75	1	1	75
2016	67	2	4	134
2019	73	5	25	365
n = 7	$\sum y = 490$	$\sum x = 1$	$\sum x^2 = 51$	$\sum xy = 108$

(2 for correct table)

The equation of the straight-line trend is

$$y_c = a + bx$$

Two normal equations are

$$\sum y = na + b\sum x$$
$$\sum xy = a\sum x + b\sum x^2$$

$\Rightarrow 490 = 7a + b$ and $108 = a + 51b$

$\Rightarrow a = 69{\cdot}9$ and $b = 0{\cdot}75$ **(1)**

$y_c = 69.9 + 0.75x$ **(1)**

Thus, trend values are

$y_{2010} = 69{\cdot}9 + 0{\cdot}75(-4) = 66{\cdot}90$

$y_{2012} = 69{\cdot}9 + 0{\cdot}75(-2) = 68{\cdot}40$

$y_{2013} = 69{\cdot}9 + 0{\cdot}75(-1) = 69{\cdot}15$

$y_{2014} = 69{\cdot}9 + 0{\cdot}75(0) = 69{\cdot}90$

$y_{2015} = 69{\cdot}9 + 0{\cdot}75(1) = 70{\cdot}65$

$y_{2016} = 69{\cdot}9 + 0{\cdot}75(2) = 71{\cdot}40$

(1 for correct trend values)

$y_{2019} = 69{\cdot}9 + 0{\cdot}75\ (5) = 73{\cdot}65$

35. Define Compound Annual Growth Rate (CAGR) and give the formula for calculating CAGR. Using the formula, calculate CAGR of Vikas's investment given below :

Vikas invested ₹ 10,000 in a stock of a company for 6 years. The value of his investment at the end of each year is given below :

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
₹ 11,000	₹ 11,500	₹ 11,650	₹ 11,800	₹ 12,200	₹ 14,000

[Use $(1{\cdot}4)^{1/6} = 1{\cdot}058$]

Sol. CAGR is the mean annual growth rate of an investment over a specified period of time longer than one year. **(1)**

$$\text{CAGR} = \left[\frac{\text{Ending investment amount}}{\text{Start amount}}\right]^{\frac{1}{\text{no.of years}}} - 1$$ **(1)**

P.V. = ₹ 10,000

F.V. = ₹ 14,000 **(1)**

n = 6 years

So, CAGR $= \left(\frac{14000}{10000}\right)^{1/6} - 1 = (1{\cdot}4)^{1/6} - 1$ $(\frac{1}{2})$

$= 1{\cdot}058 - 1$ $(\frac{1}{2})$

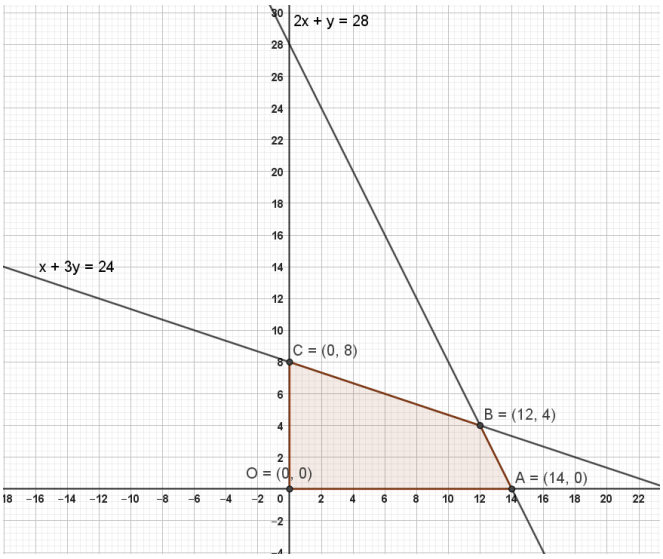
$= 0{\cdot}058$ $(\frac{1}{2})$

Hence, CAGR = 5·8% $(\frac{1}{2})$

SECTION E

This section comprises of 3 case-study based questions of **4 marks each.**

36.	A factory produces bulbs, of which 6% are defective bulbs in a large bulk of bulbs. Based on the above information, answer the following questions : (i) Find the probability that in a sample of 100 bulbs selected at random, none of the bulbs is defective. (Use : $e^{-6} = 0{\cdot}0024$) (ii) Find the probability that the sample of 100 bulbs has exactly two defective bulbs. (iii) (a) Find the probability that the sample of 100 bulbs will include not more than one defective bulb. **OR** (iii) (b) Find the mean and the variance of the distribution of number of defective bulbs in a sample of 100 bulbs.	
Sol.	n= 100, p = $\frac{6}{100}$, m= np Here m = $100 \times \frac{6}{100} = 6$. $P(r) = e^{-m}\frac{m^r}{r!}$ (i) $P(0) = e^{-m}\frac{m^0}{0!} = e^{-6} = 0{\cdot}0024$ (ii) $P(2) = e^{-m}\frac{m^2}{2!} = e^{-6} \times \frac{36}{2} = 0.0432$ (iii)(a) $P(0) + P(1) = e^{-6} + e^{-6}\frac{m^1}{1!} = e^{-6} + 6e^{-6} = 7e^{-6} = 0{\cdot}0168$ **OR** (iii)(b) Mean = Variance = m = np = 6	**(1)** **(1)** **(1+1)** **(1+1)**

37.	A factory manufactures tennis rackets and cricket bats. A tennis racket takes $1\frac{1}{2}$ hours of machine time and 3 hours of craftsmanship in its making; while a cricket bat takes 3 hours of machine time and 1 hour of craftsmanship. In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsmanship. Profit on a racket and on a bat are ₹ 20 and ₹ 10 respectively. Based on the above information, answer the following questions : (i) If x and y are the numbers of bats and rackets manufactured by the factory, then write the expression of total profit. (ii) Write the constraint that relates the number of craftsmanship hours. (iii) (a) Determine the maximum profit (in ₹) earned by the factory. **OR** (iii) (b) How many bats and rackets respectively, are to be manufactured to earn maximum profit ?	
Sol.	(i) Z = 10x + 20y (ii) $x + 3y \leq 24$ (iii) (a) other constraints are $2x + y \leq 28$ $x \geq 0$ $y \geq 0$  	**(1)** **(1)** **(1)**

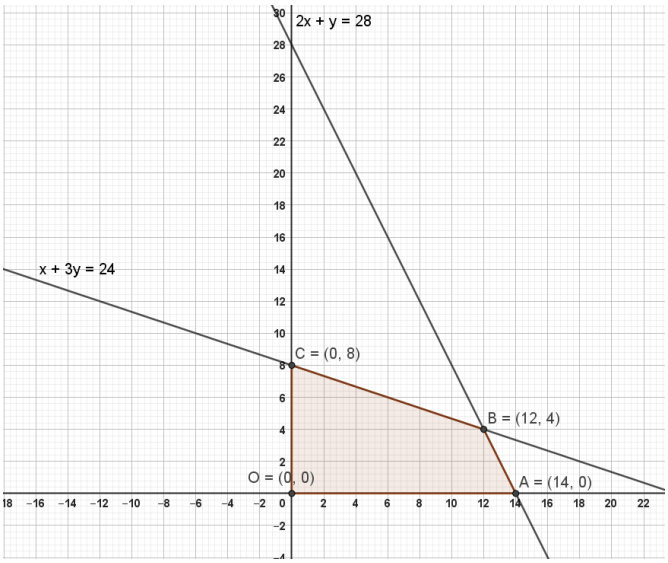
Corner Points	Value of Z
O (0,0)	0
A (14,0)	140
B (12,4)	200 → Max value
C (0,8)	160

(1)

∴ P is maximum at B (12,4); which is ₹ 200

OR

(iii) (b)



(1)

Corner Points	Value of Z
O (0,0)	0
A (14,0)	140
B (12,4)	200 → Max value
C (0,8)	160

(1)

12 bats and 4 rackets

38. In the year 2010, Mr. Aggarwal took a home loan of ₹ 30,00,000 from State Bank of India at 7·5% p.a. compounded monthly for 20 years.

Based on the above information, answer the following questions :

(i) Determine the EMI.

(ii) Find the principal paid by Mr. Aggarwal in the 150th instalment.

(iii) (a) Find the total interest paid by Mr. Aggarwal.

OR

(iii) (b) How much was paid by Mr. Aggarwal to repay the entire amount of home loan ?

[Use $(1{\cdot}00625)^{240} = 4{\cdot}4608$; $(1{\cdot}00625)^{91} = 1{\cdot}7629$]

Sol. Given P = ₹ 30,00,000, $i = \frac{7{\cdot}5}{1200} = 0{\cdot}00625$

and $n = 12 \times 20 = 240$ months

(i) $EMI = \frac{P\,i}{1-(1+i)^{-n}}$

$= \frac{30,00,000 \times 0{\cdot}00625}{1-(1{\cdot}00625)^{-240} - 1}$ $(\frac{1}{2})$

$= \frac{30,00,000 \times 0{\cdot}00625 \times 4{\cdot}4608}{3{\cdot}4608}$ $(\frac{1}{2})$

= ₹ 24167.82

(ii) Interest paid on 150th instalment

$= \frac{EMI \times [(1+i)^{240-150+1} - 1]}{(1+i)^{240-150+1}}$

$= \frac{24167 \times [1{\cdot}7629 - 1]}{1{\cdot}7629}$ $(\frac{1}{2})$

= ₹ 10458.70

⇒ Principal paid in 150th instalment = EMI – interest

=₹ (24167.82 – 10458.70) $(\frac{1}{2})$

	= ₹ 13709.12 (iii) (a) Total Interest paid = n × EMI – P = ₹ (240 × 24167.82 – 30,00,000) = ₹ 28,00,276.80 **OR** (iii) (b) Total amount paid = n x EMI = 240 x 2416.81 = ₹ 5800276.8	**(1)** **(1)** **(1)** **(1)**